EECS 861
Homework 7

1. Determine whether the following functions can be the autocorrelation function for a WSS real values random process (YES or NO):

   a) $\delta(\tau) - \sin(2000\pi\tau)$
   b) $\Lambda(\frac{\tau+1}{0.1})$
   c) $e^{-2|\tau|}$
   d) $rect(\tau / 0.5)$
   e) $\Lambda(\tau/0.5)$
   f) $u(\tau-2)e^{-\tau}$
   g) $\mathrm{sinc}(10\tau)$

2. X(t) and Y(t) are wide sense stationary, independent, zero mean, and jointly Gaussian random processes
   $Z(t) = X(t)\cos(2\pi f_0 t)+Y(t)\sin(2\pi f_0 t)$ with f_0 a constant and $C_{XX}(t_1,t_2)= C_{YY}(t_1,t_2)= C(t_1,t_2)$

   a. Find E[Z(t)]
   b. Find $C_{ZZ}(t_1,t_2)$

3. Given Z(t) in problem 2) is Z(t) wide sense stationary (YES or NO)?

4. Find the E[X(t)] and Var[X(t)] for a wide sense stationary random process with the following autocorrelation functions:

   a. $R_{XX}(\tau) = 10 + 5e^{-.2|\tau|}$
   b. $R_{XX}(\tau) = 4\mathrm{sinc}(1000\tau)$
   c. $R_{XX}(\tau) = \dfrac{40}{(1+2\tau^2)}$
   d. $R_{XX}(\tau) = 5e^{-\pi\frac{\tau^2}{9}}$

5. X(t) is a wide sense stationary zero mean, Gaussian random processes with $R_{XX}(\tau)=\Lambda(\tau)$.
    a. Find E[X(0.1)] and Var[X(0.1)], E[X(0.6)] and Var[X(0.6)]
    b. What is the distribution of X(0.1)?
    c. Find P(X(0.1)>1)
    d. What is the covariance matrix for X(0.1) and X(0.6)?
    e. What is the joint distribution of X(0.1) and X(0.6)?
    f. Find P(X(0.6)>1|X(0.1)=1)
    g. Find P(X(5)>1|X(0.1)=1)

6. X(t) is a wide sense stationary zero mean, Gaussian random processes with $R_{XX}(\tau)=\Lambda(\tau)$.
   $Z(t) = cX^2(t)$, a square law detector with constant c.
    a. Find E[Z(t)]
    b. Find $R_{ZZ}(t_1,t_2)$
    c. Is Z(t) a wide sense stationary random processes (YES or NO)?
    d. Is Z(t) a Gaussian random processes (YES or NO)?

    Hint: If X and Y are jointly Gaussian random variables then
    $E[X^2Y^2]= E[X^2]E[Y^2]+2\,(E[XY])^2$